Math2310 - Fall '22

Syllabus - Lecture 15

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Review

1 Optimization on the interior

- `defn` interior points
- `defn` critical points
- `defn` local extrema
- `thm` interior local extrema are critical points
- `thm` partial derivatives of smooth functions commute
- `defn` the Hessian: the matrix of second derivatives

Topics

1 Optimization on the interior

- the second derivative test: motivation
- an example of a function with prescribed value, gradient, and hessian at a point
  - the function $f(x,y) = \frac{1}{2}(ax^2 + 2bxy + cy^2)$ and the various shapes of its graph
  - saddles
  - paraboloid up
  - paraboloid down
  - cylindrical paraboloid
- `defn` principal curvature values and principle curvature directions of the graph $\frac{1}{2}(ax^2 + 2bxy + cy^2)$
- `thm` The determinant of the Hessian is the product of the two principle curvature directions
- consequences:
  - using the Hessian D^2f to determine the shape of the graph of $f(x,y) = \frac{1}{2}(ax^2 + 2bxy + cy^2)$
- `defn` definite, indefinite, semidefinite matrices

1.1 The second derivative test:

- adaptation of the above to study critical points of functions
- counterexamples when matrices are semi-definitite.

2 Optimization on the boundary - parametric aproach

- parameterizing the boundary of a domain and optimizing in lower dimensions.
- `exmpl` $f(x,y)=xe^y$ on $\left\{\begin{pmatrix} x \\ y \end{pmatrix} : x \geq 1, y \geq \frac{1}{2}, xy \leq 2\right\}$

3 Optimization on the boundary - Lagrange multipliers

- directions of gradients and boundary tangent directions
- `defn` constraints
- optimization under constraints
- the method of Lagrange multipliers
  - motivation
  - method of Lagrange multipliers
  - the role of the multiplier λ
  - `exmpl` $f(x,y)=xe^y$ on $\left\{\begin{pmatrix} x \\ y \end{pmatrix} : x \geq 1, y \geq \frac{1}{2}, xy \leq 2\right\}$
  - shortcomings: once candidate is found, no "second derivative test" is available

References

Textbook

- [Ste] Chap 14.7 (complete) - Maximum and minimum values
- [Ste] Chap 14.8 (complete) - Lagrange multipliers.

Videos

- The Hessian matrix | Multivariable calculus | Khan Academy - YouTube
- Multi-variable Optimization & the Second Derivative Test - YouTube
- Lagrange multipliers, using tangency to solve constrained optimization - YouTube
- Lagrange Multipliers | Geometric Meaning & Full Example - YouTube
- Lagrange Multipliers - YouTube

Geogebra applets

- Second derivative test - classification of quadratic forms - GeoGebra