Math2310 - Fall '22

Syllabus - Lecture 09 [subject to change]

by Gennady Uraltsev

Review

1 Position, velocity, and acceleration.

- exmpl Given velocity and acceleration graphically determine whether one is accelerating or slowing down (2D and 3D). Determine if one is turning.
- exmpl Given velocity and acceleration numerically determine whether one is accelerating or slowing down. (2D and 3D). Determine if one is turning.
- exmpl Given position and velocity, determine approximate position after $\mathrm{dt} = 0.1$
- exmpl Given position at t and $t + dt$, determine approximate velocity at t.
- exmpl Given velocity at $t - \mathrm{dt}$ and $t + dt$, determine approximate acceleration at t.
- exmpl Given position at $t - \mathrm{dt}$ and $t + dt$, $t + \mathrm{dt}$, determine approximate acceleration at t.
- exmpl Given position and velocity at t, determine whether the path is getting farther or closer to the origin.
- exmpl Given position on a plane $\mathcal{P}$ and velocity at t, determine on which side the path will be shortly after.

Topics

1 Approximating quantities using rate of change

- Approximate change of position given velocity
- The error estimate
  - numerical simulation
  - using FTC twice to estimate error

2 Kepler's laws

- Given a central force the quantity $\dot{\vec{p}}(t) \times \vec{p}(t)$ is constant
- Geometric meaning of the vector $\vec{N} = \dot{\vec{p}}(t) \times \vec{p}(t)$
- Motion lies in the plane through the origin, orthogonal to $\vec{N}$

3 Estimating positions at small time increments

- FTC twice: Taylor's formula.

4 Arc length

- `defn` Arc length - the length of a path without backtracks
- geometric visualization of arc length
- `thm` The arc length formula:

$$L = \int_0^T \|\dot{\vec{p}}(t)\| \mathrm{d}t$$

- geometric interpretation of the arclength formula

5 Reparametrization

- relationship between paths as functions and curves as geometric objects.
- `defn` reparameterization $t = \phi(\tau)$
- Arc length is invariant w.r.t reparameterization
- arc length parameterization

References

Videos

Textbook

- [Ste] Chap 13.4 complete (only first Kepler's law - plane of rotation does not change)
- [Ste] Chap 13.3 complete (skip Curvature and Binormal vectors. Not covered by this course)

Additional material