

Math2310 - Fall '22

Syllabus - Lecture 03

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Review

- Planes in \mathbb{R}^3 - equation form [ii]:

$$P = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : A_1x + A_2y + A_3z = B \right\}$$

- The vector $\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$ is orthogonal to the plane
- The scalar B : moves the plane away from the origin

Topics

1 The dot product [2,3]

1.1 Review

- Algebraic expression and properties.
- Geometric expression.
- defn length/norm/magnitude
- defn the word *orthogonal*.

1.2 Properties

- prop Cauchy-Schwarz inequality
- When is the dot product maximum?
- When is it 0?
- When does equality in Cauchy-Schwarz hold?
- When is the dot product positive/negative?

2 The cross product [3,4]

- Algebraic expression and properties.
- Geometric properties.
- Using the cross product to find an orthogonal vector

References

Textbook

- [Ste] Chap 12.3 The Dot Product pp847 - 854
- [Ste] Chap 12.5 Equations of Lines and Planes pp864 - 870
- [Ste] Chap 12.4 The Cross product pp855-864

Videos

1. Vector dot product and vector length | Vectors and spaces | Linear Algebra | Khan Academy - YouTube
2. Geometrically Defining the Cross Product | Multivariable Calculus - YouTube
3. 30.2 Cross Product - YouTube
4. 30.3 Cross Product in Cartesian Coordinates - YouTube
5. Find the projection of Two Vectors - YouTube
6. Projection of point onto a plane - YouTube

Additional material

Videos

- i. Calculus III: The Cross Product (Level 1 of 9) | Geometric Definition - YouTube
- ii. Defining a plane in \mathbb{R}^3 with a point and normal vector | Linear Algebra | Khan Academy - YouTube
- iii. Deriving the law of cosines from the dot product by applying distributive and commutative properties - YouTube
- iv. The determinant | Chapter 6, Essence of linear algebra - YouTube

Geogebra applets

- Dot Product Insight – GeoGebra
- 3. The Dot Product and Projections – GeoGebra
- Geometry of Cross Product – GeoGebra Visualisation of Vector (Cross) Product – GeoGebra
- Geometric Interpretation of the Cross Product – GeoGebra