Math2310 - Fall '22

Syllabus - Lecture 01

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Review

First day of class :) Welcome!

Topics

1 What are vector spaces and vectors?

- What are vectors and vector spaces?
- algebraic representation of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$: tuples of real numbers.
- visual representation of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$: points and/or arrows.
- `rmk` Right-handedness/orientation of representation in $\mathbb{R}^2$ and $\mathbb{R}^3$.
- `rmk` Do not use two arrows on coordinate axis!
- `defn` the meaning of the words: *vector space*, *point*, *component*, *dimension*, *projection*.
- `defn` scalars.
- the vectors $\vec{0}$ in all dimensions: `defn` the *origin*.
- `notation`
  - Vectors are denoted with an arrow or with boldface: $\vec{u}$ or $\boldsymbol{u}$.
  - Coordinate representation of vectors is given by row or column vectors:

    $\vec{u} = (1, 2, 3)$ or $\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.
  - Components are represented by indexes: $\vec{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$:

    `exmpl` if $\vec{u} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ then $u_1 = 3$, $u_2 = 4$, $u_3 = 5$.

2 Operations on vectors: addition, multiplication by scalars

- algebra of vectors:
  - addition
  - multiplication by scalars
  - factoring out scalar multiples

- graphical visualization of algebra of vectors
    - addition corresponds to tail to end.
    - multiplication by scalars correspond to scaling/zooming
    - addition corresponds also to translation
- coordinates and basis vectors:
    - defn standard basis: the vectors $\hat{e}_1$, $\hat{e}_2$ of $\mathbb{R}^2$ and the vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ of $\mathbb{R}^3$
    - defn representing a vector in the standard basis (briefly mentioned in class, [Ste] pp 841)
        - the relation of the coordinates of the vectors with their standard basis representation
    - que What is a good notion for "a collection of basis vectors in $\mathbb{R}^3$"? (not discussed in course yet)

- Notation for sets
    - notation representing sets using curly braces: for example

$$A = \{(x, y, z) \in \mathbb{R}^3 : x > y\}$$

      or equivalently

$$A = \left\{\vec{u} \in \mathbb{R}^3 : u_1 > u_2\right\}$$

    - notation representing sets using equation form: for example

$$A = \left\{(x, y, z) \in \mathbb{R}^2 : x^2 + y^2 = 1\right\}$$

      or equivalently

$$A = \left\{\vec{u} \in \mathbb{R}^2 : u_1^2 + u_2^2 = 1\right\}.$$

    - notation representing sets using parametric form: for example

$$A = \left\{(x, y, z) \in \mathbb{R}^3 : (x, y, z) = (1, 0, 0) + \lambda(1, 2, 3),\ \lambda \in \mathbb{R}\right\}$$

      or the shorter version

$$A = \left\{(1, 0, 0) + \lambda(1, 2, 3) \in \mathbb{R}^3,\ \lambda \in \mathbb{R}\right\}$$

      or even shorter

$$A = \left\{(1, 0, 0) + \lambda(1, 2, 3),\ \lambda \in \mathbb{R}\right\}$$

- Typical questions of analytic geometry (only questions, no answers given in class yet):
    - que given 3 points in $\mathbb{R}^3$ find a plane passing through them

- que given 4 points in $\mathbb{R}^3$ determine if they line in the same plane
- que given 2 points in $\mathbb{R}^2$ or $\mathbb{R}^3$ find the line passing through them
- que given 2 points in $\mathbb{R}^2$ or $\mathbb{R}^3$ determine if a third point lies on it
- que given 2 lines in $\mathbb{R}^2$ or $\mathbb{R}^3$ determine if they are parallel
- que given 2 lines in $\mathbb{R}^3$ determine if they lie in the same plane
- que what is the angle between to lines through the origin in $\mathbb{R}^2$?
- que what is the angle between to lines through the origin in $\mathbb{R}^3$?

References

Textbook

Textbook references pages in

James Stewart, Saleem Watson, Daniel K. Clegg - Multivariable Calculus, Metric Edition - 9th edition - Cengage Learning

- [Ste] Ch 12.1 pp 829-836 (complete)
- [Ste] Ch 12.2 pp 837-847 (complete)

Videos

- Representing points in 3d | Multivariable calculus | Khan Academy
- The Vector Equation of Lines | Multivariable Calculus

Additional material

Videos

- Vectors | Chapter 1, Essence of linear algebra
- Linear combinations, span, and basis vectors | Chapter 2, Essence of linear algebra

Geogebra applets

- Adding Vectors – GeoGebra
- Adding Vectors Geometrically – GeoGebra
- 3D Vector visualization – GeoGebra
- Vector Addition (3D) – GeoGebra