Math2310 - Fall '22

Midterm02 Information

by Gennady Uraltsev

Time and place

What: MATH 2310 Midterm 02

When: Thu Nov 17, 2022 18:30 (2:00 hours total + extra time for students with SDAC accommodation)

Where: CHEMICAL ENGINEERING BLDG ROOM 005

Please be in the class at 18:20. Exam starts at 18:30.

Make-up exam

What: MATH 2310 Midterm 02 make-up

When: Thu Nov 19, 2021 9:00– 11:00(EDT) (2:00 hours total)

Where: TBA

Please be in the class at 08:50. Exam starts at 9:00

Format

The exam will be an open book written test with multiple parts.

You will be provided a printed out booklet with the problems where you will need to write your solutions and show your work. You can use additional draft paper that you can attach to the booklet.

You have 120 minutes to work on the exam.

If any issues arise during the exam, please let us know immediately by raising your hand.

If you finish early you may hand in your material and leave. You may NOT leave during the last 15 min of the exam.

Discussing the exam publicly or with people that have not taken the exam is strictly forbidden until

Morning of Saturday, Nov 20, 2022.

Allowed material

The exam is open book. The material you may consult during the exam is:

- course textbook
- any material posted on official course channels
    - Collab syllabus

    - solutions of HW on Collab
    - Piazza questions or answers(saved in offline form)
- personal notes
- An offline installation of Geogebra 5 or Geogebra 6 or graphing calculator.

All material must be offline. Internet access is forbidden during the midterm.

NO OTHER MATERIAL IS PERMITTED

You may have only 1 electronic device (tablet/laptop, **no phones**) on which you may consult your notes or you may print notes out and use a calculator suite. This device must be in airplane mode. You must make sure all other software is turned off. Only the pdf/note reader software must be open an CAS may be open. Make sure to turn off: *email client, chat software, music player, etc.*

- Most problems require justification.
- Merely providing a correct answer affords partial credit. Adding a picture may help but is still not a full answer you will not be able to print or attach screenshots so be able to do basic sketches by hand.
- Full credit requires full algebraic justification.

Any violation of this policy is considered an academic integrity violation, will result in immediate invalidation of the exam, and will be escalated appropriately.

Topics and review suggestions.

All topics covered in class/recitation and assigned as HW by 2022-09-29 is possibly material of exam.

Non-exhaustive list of topics

Partial derivatives and gradients, especially of functions of 3 variables

- Review of functions $f:\mathbb{R}^2\to\mathbb{R}$
    - Level sets of functions $f:\mathbb{R}^2\to\mathbb{R}$
    - graphs of functions $f:\mathbb{R}^2\to\mathbb{R}$
- functions $f:\mathbb{R}^3\to\mathbb{R}$ and their level sets
    - Level sets of functions $f:\mathbb{R}^3\to\mathbb{R}$
    - Examples of functions $f:\mathbb{R}^3\to\mathbb{R}$ and their level sets: functions whose level sets are sphere, cylinders
- computing partial derivatives
- using partial derivatives to linearize/approximate functions $f:\mathbb{R}^3\to\mathbb{R}$
- partial derivatives of functions of $f:\mathbb{R}^3\to\mathbb{R}$

- the gradient vector
  - definition and relation to partial derivatives
  - geometric representation of gradient of $f:\mathbb{R}^3\to\mathbb{R}$ in $\mathbb{R}^3$
  - meaning of direction of gradient vector in $\mathbb{R}^3$
  - meaning of magnitude of gradient vector in $\mathbb{R}^3$
  - relation between direction of gradient vector and direction (tangent) of level surfaces
  - behavior of level surfaces around points where gradient is non-vanishing
- tangent plane to a level set of a function $f:\mathbb{R}^3\to\mathbb{R}$: equation and parametric form
- find a (or two non parallel) tangent vectors to the level set of a function $f:\mathbb{R}^3\to\mathbb{R}$
- the normal line to a level set of a function $f:\mathbb{R}^3\to\mathbb{R}$: parametric form
- the tangent plane to a graph of a function $f:\mathbb{R}^3\to\mathbb{R}$: equation form and parametric form
- the normal line to a graph of a function $f:\mathbb{R}^3\to\mathbb{R}$
- The norm function $f(\vec{v})=\|\vec{v}\|$, its gradient, its level sets
- Chain rule for multivariable functions $\frac{\mathrm{d}}{\mathrm{d}t}f(\vec{p}(t))$
- FTC for $f:\mathbb{R}^3\to\mathbb{R}$ along paths $\vec{p}:\mathbb{R}\to\mathbb{R}^3$

Shapes of domains

- domains in $\mathbb{R}^2$ and $\mathbb{R}^3$
  - closedness
  - interior points
  - boundary points
  - boundness

Optimization

- functions and their extrema
  - bounded functions
  - <u>`defn`</u> global max & global min
  - <u>`defn`</u> local max & local min
- quantifiers in the above expression
- <u>*`thm`*</u>: continuous function on closed and bounded domain is bounded and attains global max and global min

Optimization in the interior points

- `defn` Critical points in the interior of the domain
- the Hessian and the second derivative test:
  - geometric meaning of the Hessian
  - principle curvature values and their relation to the determinant of the Hessian
  - positive/negative/semi-positive/semi-negative definiteness; indefiniteness
  - how level curves look around a critical point where the Hessian is positive definite, negative definite, indefinite.
  - examples of functions with prescribed Hessian
  - the second derivative test
- second partial derivatives commute

Optimization on the boundary

- qualitative behavior on the boundary: direction on the gradient and shape of the domain.
  - conditions on the gradient that preclude being a local max / min
- optimization on the boundary: parameterization of the boundary
- optimization on the boundary: Lagrange multipliers
  - qualitative reasoning about Lagrange multipliers
  - setting up and solving Lagrange multipliers
- exceptions to the Lagrange multipliers: critical points on the boundary

Integration

- Integrals in higher dimension: meaning
- Riemann sum approximation
- properties of integrals: monotonicity, linearity, additivity under disjoint unions of domains

Standard integration techniques

- Fubini in 2D
  - applications of Fubini in 2D: computing integrals iteratively
  - switching limits of integration in 2D
  - deciding on how to order integration variables
  - splitting domains to set up integrals
- describing domains in $\mathbb{R}^2$ to apply Fubini:
  - integrating in x the slices in y and integrating in y the slices in x

- Fubini in 3D
  - applications of Fubini in 3D: computing integrals iteratively
  - switching limits of integration in 3D
  - Fubini 1 step at a time: integrating in z the slices in x, y (and similar)
- Areas and volumes: integrating the function 1.

Using symmetries

- Symmetries of integrals: mirroring, translation, rotations
- relation to symmetries of domain and of function
- cancellation: oddness
- stretching: linear, area, and volumetric scaling
- example: integral of ellipse

Changes of variable formulas

- Change of variables formula in 2D and 3D: generalities.
- geometric interpretation of the change of variables formula
- The det of the Jacobian as the stretch factor.
- The determinant as a volume of a parallelipiped
- Computing the determinant of 2×2 and 3×3 matrixes
- Change of variables formula in 2D: establishing the bounds of the domain, setting up the integral, computing the integral
- special changes of variables: linear transformations, translations ($\mathbb{R}^2$ or $\mathbb{R}^3$)
- geometric meaning of the vectors $\partial_u \Phi(u, v, w)$, $\partial_v \Phi(u, v, w)$, $\partial_w \Phi(u, v, w)$
- geometric meaning of the vectors $\partial_u \Phi(u, v)$, $\partial_v \Phi(u, v)$.
- Identifying the above vectors on a graphical prepresentation given the explicit formula for $\Phi(u, v)$

Special changes of variable formulae

- linear, polar, spherical, cylindrical coordinates
- finding bounds of domains in linear, polar, spherical, cylindrical coordinates
- setting up and computing integrals in the above coordinates
- simple modifications of changes of variables, for example:
  - cylindrical coordinates with the cylinder along the x axis,

- cylindrical coordinates with the cylinder along the y axis,

- spherical coordinates centered at $\vec{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

Surface integrals

- Parameterized surfaces
- the parameterization of a surface $\Phi : \mathcal{D} \to \mathbb{R}^3$
- the geometric meaning of the vectors $\partial_u \Phi(u, v)$ and $\partial_v \Phi(u, v)$ (slightly different from the 2D or 3D) change of variables formula
- recognizing $\partial_u \Phi(u, v)$ and $\partial_v \Phi(u, v)$ from the algebraic expression of Φ and a graphical representation
- Finding the tangent plane to a parameterized surface at a point
- Finding the normal line to a parameterized surface at a point
- setting up surface integrals, the surface area.
- the meaning of the term $\|\partial_u \Phi \times \partial_v \Phi\|$
- surface are of graphs of functions
- using analogues of cylindrical or spherical coordinates to describe surfaces

Vector fields

- definition, representation, and examples of vector fields in 2D
- gradient vector fields (conservative vector fields): definition
- gradient vector fields have vanishing curl: $\partial_x F_y - \partial_y F_x = 0$

Path integrals

- definition of path integral: computing it analytically
- graphical intepretation of path integrals
- properties of path integrals:
  - linearity in the field
  - additivity w.r.t. concatenation of paths
  - invariance w.r.t. reparameterization
  - change of sign when backtracking.
- path integrals as Riemann sums: approximation

$$\int \vec{F} \cdot \mathrm{d}\vec{p} \approx \sum_i \vec{F}(\vec{p}(t_i))(\vec{p}(t_{i+1}) - \vec{p}(t_i))$$

i.e. sums of dot products of the field with steps

- graphical interpretation of path integrals
- estimating path integrals using Riemann sums
- estimating and interpreting the sign of path integrals
- path integrals and conservative vector fields:
  - computing path integrals knowing the potential: FTC for conservative vector fields
  - If $\vec{F}$ is conservative then the path integral depends only on start point and end point, not on the path in between
- Recovering potentials of conservative vector fields:
  - Using 1D FTC twice (as in recitation)
  - Using path integrals is NOT ON THE MIDTERM02
- Excluding the possibility of a vector field being conservative based on graphical representation