Math2310 - Fall '22

Midterm01 Information

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Time and place

What: MATH 2310 Midterm 01

When: Thu Oct 06, 2022 18:30–20:30(EDT) (1:30 hours total + extra time for students with SDAC accommodation)

Where: CLARK HALL ROOM 107

Please be in the class at 18:20. Exam starts at 18:30.

Make-up exam

What: MATH 2310 Midterm 01 make-up

When: Fri Oct 07, 2021 9:00– 10:00(EDT) (1:30 hours total + extra time for students with SDAC accommodation)

Where: TBA

Please be in the class at 08:50. Exam starts at 9:00

Format

The exam will be an open book written test with multiple parts.

You will be provided a printed out booklet with the problems where you will need to write your solutions and show your work. You can use additional draft paper that you can attach to the booklet.

You have 90 minutes to work on the exam.

If any issues arise during the exam, please let us know immediately by raising your hand.

If you finish early you may hand in your material and leave. You may NOT leave during the last 15 min of the exam.

Discussing the exam publicly or with people that have not taken the exam is strictly forbidden until

Morning of Saturday, Oct 8, 2022.

Allowed material

The exam is open book. The material you may consult during the exam is:

- course textbook

- any material posted on official course channels
    - Collab syllabus
    - solutions of HW on Collab
    - Piazza questions or answers(saved in offline form)
- personal notes
- An offline installation of Geogebra 5 or Geogebra 6 or graphing calculator.

All material must be offline. Internet access is forbidden during the midterm.

NO OTHER MATERIAL IS PERMITTED

You may have only 1 electronic device (tablet/laptop, **no phones**) on which you may consult your notes or you may print notes out and use a calculator suite. This device must be in airplane mode. You must make sure all other software is turned off. Only the pdf/note reader software must be open an CAS may be open. Make sure to turn off: *email client, chat software, music player, etc.*

- Most problems require justification.
- Merely providing a correct answer affords partial credit. Adding a picture may help but is still not a full answer you will not be able to print or attach screenshots so be able to do basic sketches by hand.
- Full credit requires full algebraic justification.

Any violation of this policy is considered an academic integrity violation, will result in immediate invalidation of the exam, and will be escalated appropriately.

Topics and review suggestions.

All topics covered in class/recitation and assigned as HW by 2022-09-29 is possibly material of exam.

Non-exhaustive list of topics

Vector operations

- addition and multiplication by scalar: algebra and geometry
- dot product and its properties:
    - algebraic formula
    - geometric formula
    - when it is maximum/minimum/zero
    - Cauchy-Schwarz inequality

- cross product and its properties:
    - algebraic formula
    - geometric formula
    - when it is maximum/minimum/zero
- angles: using dot or cross products to determine angles
- magnitude: formula and geometric meaning
- law of cosines: generalized Pythagoras' theorem
- computing areas: using cross product
- computing volumes: triple product formula:

Analytic geometry

- Lines in $\mathbb{R}^2$: parametric and equation form
    - Line through two points in $\mathbb{R}^2$: parametric form
    - Line through two points in $\mathbb{R}^2$: equation form
    - Line through a point in $\mathbb{R}^2$ and containing a direction
    - Lines in $\mathbb{R}^2$: passing from parametric to equation
    - Lines in $\mathbb{R}^2$: passing from equation to parametric
    - Lines in $\mathbb{R}^2$: geometric meaning of coefficients appearing in equation form (normal direction, displacement from origin)
    - Computing angle between two lines in $\mathbb{R}^2$
    - Decide whether 3 points lie on the same line + quick method using cross product
    - Given line and two points not on the line decide whether they are on same or opposite sides
- Planes in $\mathbb{R}^3$: parametric and equation form
    - Plane through 3 points in $\mathbb{R}^3$: parametric form
    - Plane through 3 points in $\mathbb{R}^3$: equation form
    - Planes containing a point and two directions
    - Planes in $\mathbb{R}^3$: passing from parametric to equation
    - Planes in $\mathbb{R}^3$: passing from equation to parametric
    - Planes in $\mathbb{R}^3$: geometric meaning of coefficients appearing in equation form (normal direction, displacement from origin)

    - Computing angle between two planes in $\mathbb{R}^3$
    - Decide whether 4 points lie on the same plane + quick method using cross product
    - Given plane and two points not on the plane decide whether they are on same or opposite sides
- projections
    - scalar and vector projections
    - projections of point onto line through $(0,0)$ in $\mathbb{R}^2$ or $\mathbb{R}^3$: formula and geometric meaning
    - projections of point onto line NOT through $(0,0)$ in $\mathbb{R}^2$ or $\mathbb{R}^3$: formula and geometric meaning
    - projections of point onto plane through $(0,0)$ in $\mathbb{R}^3$: formula and geometric meaning
    - projections of point onto plane NOT through $(0,0)$ in $\mathbb{R}^3$: formula and geometric meaning
- Orientation
    - on $\mathbb{R}^2$: clockwise and counterclockwise rotation and relation with cross product
    - on $\mathbb{R}^3$: direction of cross product and right-hand rule.
- orthonormal bases
    - definition on $\mathbb{R}^2$ and $\mathbb{R}^3$
    - checking whether an orthonormal basis is correctly oriented on $\mathbb{R}^2$ and on $\mathbb{R}^3$
    - given a point in standard coordinates, express it w.r.t. a given orthonormal basis.
- polar coordinates

Paths and vector functions

- definition of path & examples
- expressing paths using polar coordinates:
    - rotation direction
- visualizing paths
- domain limitations when expressing paths
- operations on paths:
    - checking whether a path lies on a set (planes, surface)
    - projecting a path means projecting the position point at every time.

- differential calculus of paths:
    - velocity and derivatives of paths
    - speed
    - acceleration
    - rules of differentiation of vector expressions depending on t:
        - Leibniz rule (aka product rule): derivative of dot product, derivative of cross product, derivative of product of scalar function by vector function
        - chain rule for paths and reparameterization
    - derivative of the magnitude of a vector function:
        - formula
        - geometric interpretation
- FTC for 1D functions (review)
- FTC for paths
- Linearization of vector functions:
    - tangent line at fixed time
    - approximation of position based on position at close time and velocity

Dynamics of paths

- graphical interpretation and representation of velocity and acceleration
- velocity vector and speed:
    - velocity vector
    - unit tangent vector
    - rates of change of distances
    - tangent line to a path
    - speed
- acceleration vector
    - tangential and normal acceleration
    - effect on speed and direction
    - unit normal vector

- approximating change of velocity and speed based on acceleration and its components

Improtant examples of motion

- uniform circular motion
- motion of projectile under vertical gravitational pull.

Length and reparameterization

- Lengths of paths
- reparameterization of paths: defintion
- reparameterization of paths: geometric meanin
- lengths are reparameterization invariant
- formulas related to reparameterization: consequences of chain rule:
    - velocity of reparameterized paths
    - speed of reparameterized paths
- arclength reparameterization
    - properties of arclength reparameterization
    - finding arclength reparameterization using the arc length function
    - the arclength ODE

Functions of multiple real variables

Defintions and representation

- definition, domains, examples
- representing functions 2 real variables:
    - graphs
    - contour plots
    - contour plots as intersection of graph with plane
- properties of level sets: shape and intersection
- associating functions, graphs, and level sets; reading contour plots
- freezing one variable of a function and using single variable techniques

Partial derivatives and gradients

- computing partial derivatives
- using partial derivatives to linearize/approximate functions.
- partial derivatives of functions of 2 real variables: slopes of graphs of slices
- the gradient vector
    - definition and relation to partial derivatives
    - geometric representation of gradient
    - meaning of direction of gradient vector
    - meaning of magnitude of gradient vector
    - relation between direction of gradient vector and direction (tangent) of level curves